5543/MP1 **MAY 2006**

Paper I — MATHEMATICAL PHYSICS

Time : Three hours Maximum : 100 marks

Answer ALL questions choosing either (a) or (b) from each.

All questions carry equal marks.

1. (a) State and prove Stoke's theorem in space.

Or

(b) Define covariant, contravariant and mixed tensors. Explain the contraction of a tensor. Define metric tensor.

2. (a) Find a series of sines and cosines of multiples of x which will represent $f(x)$ in the interval $-\pi \leq x \leq \pi$ when $f(x) = 0, \quad -\pi \leq x \leq 0$

$$= \frac{1}{4}\pi x, \quad 0 < x < \pi$$

Hence or otherwise deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

Or

(b) (i) Show that $\beta(m,n)=\beta(n,m)$

(ii) Show that

$$m\,\beta(m,n+1)=n\,\beta(m+1,n)$$

(iii) Find the value of $\Gamma(\frac{1}{2})$ and hence deduce the value of $\Gamma(-\frac{1}{2})$ and $\Gamma(-\frac{3}{2})$.

3. (a) Evaluate the following integrals

(i) $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\,d\theta$

(ii) $\int_0^{\infty} \frac{dx}{x^6+1}$

Or

(b) (i) Find the Fourier transform of the function

$$f(x)=\frac{a}{x^2+a^2} \text{ for } a>0.$$

(ii) Show that the Fourier transform of a Gaussian probability function is also a Gaussian probability function.

4. (a) Solve the Bessel differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ and obtain the expression for $J_n(x)$.

Or

(b) Starting from the generating function for Hermite polynomials prove the following recurrence relations

(i) $2\,n\,H_{n-1}(x)=H_n'(x)$

(ii) $2\,x\,H_n(x)=2\,n\,H_{n-1}(x)+H_{n+1}(x)$.

5. (a) Set up the wave equation for a stretched string and obtain its solutions under certain initial conditions.

Or

(b) Obtain the Greens function for Poisson equation.